Strand: Geometry and Measurement – Measurement

Level: NZC 4

Achievement objective: Use side or edge lengths to find perimeters and areas of rectangles parallelograms, and triangles and the volumes of cuboids

LI: Finding the area in right and non-right-angled triangles

Key competencies:

* **Thinking**: Students are to be continuously thinking about their ideas and working, throughout the lessons students are able to reflect on their own learning and other students learning. This brings a cycle of continuous thinking and learning.
* **Language, symbols and text**: Mathematics offer multiple opportunities for the use of language and symbols. Students are to be using discussion in the lessons, they will be doing collaborative work with pairs or as a group. This will require usage of language on the form of everyday language and mathematical language. There is usage of symbols in the lesson, such as using order of operations.
* **Relating to others**: Students to build relationships with each other through the various amounts of peer work offered. From that students are to

Students would also know basic geometry knowledge such as what polygons and how to find an area of a rectangle, this would have been covered in a recent unit the students have done. The lessons I will be teaching will develop from their prior knowledge of finding areas of a rectangle.

**Background knowledge**

The background knowledge students have been based off the achievement objectives from NZC and the Numeracy framework books. From this I can point out that the students know all the numbers in the range 0-1,000,000; decimals to three places; symbols for any fraction including tenths, hundredths, thousandths, and improper fraction and know the number word sequences, forwards and backwards in ones, tens, hundreds and thousands, and halves quarters, thirds, fifths and tenths in fractions. Students will be able to round whole numbers to the nearest tens, hundreds and thousands and rounding decimals up to two to the nearest whole number. Basic facts wise students can know addition and subtraction facts up to 20, multiplication facts up to the 10 times table and some corresponding division facts; and multiplication facts with tens, hundreds and thousands (e.g 10x100). In this topic students, will be asked to unpack their knowledge of basic facts as they will be dealing with finding perimeter (2a + 2b) or area (a x b). Students will have the possibility of being extended when dealing with numbers larger than their current stage level. The strategies students can use are a wide range of part-whole strategies to solve and estimate answers. They are also able to solve problems from known facts (6x6 as (5x6) + 6 or 6x8 is the same as two lots of 3x8=24, so 6x8 = 24 + 24 = 48). To be concise students should know how to complete mathematics in stage 6 “Advanced Additive - Early Multiplicative” which is the stage below they are working at. In relation to the New Zealand Curriculum students are past level 3, so in relevance to the geometry and measurement strand, students should be use linear scales and whole numbers of metric units for length, area, volume and capacity, weight (mass), angle, temperature, and time. Also, able to find areas of rectangles and volumes of cuboids by applying multiplication.  This means students can know the area of a rectangle but not necessarily know it is the side of lengths multiplied.

Students are working towards or at stage 7 “Advanced Multiplicative - Early Proportional”. This means they are working towards understanding decimal word sequences forwards and backwards, by thousandths, hundredths, tenths, ones, tens, hundreds, thousands and orders decimals to three places. Students are working towards knowing the grouping of numbers to 10 that are in numbers to 100 and be able to find the remainders (e.g. nines in 68); the grouping of 10, 100 and 1000 that can be made from a number up to seven digits (e.g. tens in 54, 521). They are also working towards equivalent fractions for halves, thirds, quarters, fifths, and tenths with denominations up to 100 and up to 1000. At stage 7 students will be able to round to whole numbers or tenths with decimals up to two places (9.46 to 9.5 (nearest tenth)). The basic facts knowledge will be divisions up to the 10 times tables and be able to convert fractions to decimals to percentages for halves, thirds, quarters, fifths, and tenths. Also stage 7 indicates they know the divisibility rules for 2, 3, 5, 9, and 10. Student identifies factors of numbers to 100, including prime numbers and common multiples of numbers to 10. This is level 4 NZC, where students instead of using linear scales they use appropriate scales (mm, cm, or m), convert between metric units, using whole numbers and decimals and use sides or edges to find perimeters of 4 sided polygons and triangles as well as volumes of cuboids.

One method of obtaining diagnostic information is through a snapshot exercise. This can be done as a worksheet or electronic form, for example a Kahoots! to bring technology and motivation into the activity as the students are 21st century learners. A snapshot is where you have various questions from different difficulty with different types of questions about the same topic. There are 3 parts to the snapshot, there is the first sheet where students are to complete various questions. Each question made are is linked to a part of an achievement objective and learning intention they have learnt in the past, and towards to future (to know students who are struggling and who need extension work), each AO is broken down into key learning intentions within the unit. This is so when it comes to the second part of marking, I can know what learning area the student needs to work on. In the picture below you can see this is done with the whole class in one page, from this you can grab a “snapshot” of the students right and wrong questions, with the help of colour coding you can have a brief idea of what are the gaps in the students learning and as a class/group as a whole. The third part is to give the students a separate piece of paper with each learning intention from the snapshot worksheet, the students can identify what LI’s they can do and what they got wrong. This piece of paper can be stated as an individual goal plan to enforce autonomy within the students learning and be able to know what they need to work towards. This will help in the sequence lessons where students will understand when and why they are differentiated into groups.

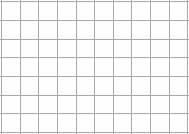
This diagnosis activity is useful for both the student and the teacher as everyone has individual LI achieve (mini goals).

**Lesson sequence**

The learning sequence builds up students’ strategy and knowledge in solving geometry problems using multiplication. Students will specifically learn the relationship between the perimeter and area of rectangles and recognize that a triangle has half the area of a rectangle with the same base and height lengths to be able to apply the rule 'area of a triangle equals half base times height. The sequence of lessons described below is following a unit on area of rectangles. Multiple scholarly articles show that students find concepts of triangles difficulty with various misconceptions can be obtain by students when geometric properties of triangles are not taught properly, because of this my lessons will strive to provide indications of possible misconceptions arising. These articles will be discussed in the misconception section.

Lesson 1 Students are dividing rectangles diagonally to produce right angled triangles, students will realize that the two triangles formed are of equal area.

Students will first begin the activity by cutting out grid paper (this is will be useful later in the lesson) into a shape of a rectangle. There will be a limit on the size of the rectangle to 10 squares per sides as this is the extent of their knowledge in basic facts times table.



Then ask the students to find the area of their rectangle (remember the units as students tend to forget). Students should be encouraged to work it out mentally and explain their methods as a quick recap of their previous lessons. Also, it will help bring students up to speed using ako. (Students should be confident in this material as they have covered this topic in the previous unit)

The next step is for the students to rule a line diagonally from one side to the other in the rectangle. This in turn will make two right angled triangles.(nzmaths.co.nz)

Get students to realize that cutting the rectangle in half will create two triangles. Ask the students to find the area of each triangle. Some students may want to count the squares in the triangle. From this we can ask “What do you notice about the areas of the two triangles?”

Students should notice that the two triangles have the same areas and each triangle is half the area of the rectangle. If students found out the area through counting, they will be asked what calculations can you check your answer?

Possible questions made from teachers:

*What is the area of each triangle exactly?*

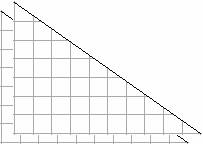
*How could you work out the areas of the triangles?* (Could you half the area of a rectangle?)

*What numbers would you need to multiply or divide?*(Base times height times half or base time half divided by two)

*What would be the easiest strategy for you?* (Students might have the misconception of having to multiply the base and height before dividing, most times the better strategy is to half either the base or height before multiplying)

(nzmaths.co.nz)

The ideas that were previously discussed can be reinforced by having students cut along the diagonal and rotate one triangle to sit on top of the other.  They will then see that not only do the triangles have the same area, but they are identical triangles.

(nzmaths.co.nz)

After you can pose the question “Will this work for any rectangle?” Students are then to investigate and make various kinds of rectangle to discover for themselves that the rule for the triangle works for any kind of rectangle. This can fix the diffused misconception among students that triangles can only have equilateral properties. Throughout the lesson there will also be multiple opportunities to address the issue of

The justification to why I have introduced triangles in this manner is for students to discover the relationships triangles have with four sided polygons. The lesson is conducted using materials, a teaching model that has been suggested in Numeracy Project Book 3

(Ministry of Education. (2007a), where introductory lessons should begin with physical materials.  This allows concrete knowledge for the students to progress to imaging then to number properties to is backed up by Gate (2001) which states “using something tangible and visible helps pupils draw connections more easily” (pg140), which is backed up further by Anthony (2007) which finds that using materials helps students strategic thinking and solution strategies. Discovery is also a more efficient way of teaching compared to just telling students how it works. Discovery leads students to help connect the dots and students learn rather memories. Also, having students to be able to cooperate and talk to each other on their ideas will help promote motivation into the students. Powell, K. C., & Kalina, C. J. (2009) shows that collaborative learning increases students motivations in the classroom which has a positive correlation to academic achievement.

Lesson 2 will be moving towards students drawing right angled triangles, completing a rectangle based on the triangle and finding the area of the original triangle. This will help reinforce the idea of last lesson and progress into finding a way to calculate the area of a triangle.

Students will have a mini whiteboard each and draw a right-angled triangle with a base of 10cm and a height of 5cm and prompt the students if they can work out its area (Reminder on units). Students are then to be asked for a think, pair, share. They are to think for 30 seconds and write down what they have thought of on their whiteboard. They are then to pair up and share afterwards (Students are told that they will be asked on what their pair said so they must pay attention). Afterwards students are to regroup have a discussion on their ideas. If students did not have the right idea, then they can be prompted with these questions:

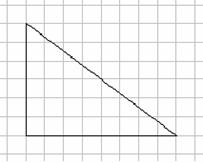
*Do we have enough information to work out the area?*

*Could we work out the area of another shape that would help?*

*Remember what we discovered about rectangles last maths lesson – could that help?*

*(nzmaths.co.nz)*

Afterwards students to draw a triangle on their grid paper so that two of its sides are along lines of the grid paper.

(nzmaths.co.nz)

This part of the lesson will allow students to visually see and prove the connection between the area of a rectangle and a triangle. Students are to draw a same triangle on the grid paper to form a rectangle, students are then challenged to find the area of the rectangle to find the area of the triangle. They will do this multiple times to ensure that they understand the connection between the two shapes. Once they have done that return as a group to have a discussion on what they just did.

Now get them to draw the matching triangle that makes a rectangle.  Draw the triangle to make a rectangle on your diagram on the board to illustrate.

Questions will be as follows

*Does this work for every right-angled triangle?*

*Can you describe a rule for the area of a right-angled triangle?*

*Are there any clever tricks to make the maths easier?* *[Divide one of the sides by two before multiplying]*

(nzmaths.co.nz)

Get students to record a rule for the area of right angled triangles in their own words, this is important as they are writing down what they think and see. Students should be able to see that the area of any right-angled triangle is equal to the area of the rectangle with the same base and height divided by two. Get them to record the statement “For right angled triangles, area equals half base times height.” Ensure that they can see that this means the same thing. This will help students into the formula for finding out an area of a triangle without explicitly stating it. Make sure to explicitly say it at the recap of the lesson, if you have a right-angled triangle

The justification for the start of the lesson is based from Schön, D. A. (1987) findings, they found that students who were not afraid of making mistakes have a higher academic achievement compared to the students who were afraid of making mistakes in the classroom. By having each student have a mini-whiteboard it removes the idea of the student’s work bring permanent. Without this idea students, will not be afraid of making mistakes in their work. This is turn makes this lesson more effective with students learning from their mistakes by developing a reflective approach to their thinking, students look back on their work and would think why or how their answer was no correct, they will look towards other people’s thinking and develop their ideas on those. This lesson is a progression of the lesson in regards to the Numeracy Development strategy theory. Students are being connected towards last lesson and how they can find the area of a triangle using imaging. My lesson still steams from the discussion by Gates (2001) where “using something tangible and visible helps pupils draw connections more easily”.

Lesson 3 will be dealing with misconceptions of finding the height of a non-right angled triangle and leading to finding the area of non-right angled triangles

Students will be asked to investigate with a pair to find the area of an unlabeled non right angled triangle. Students will be asked on what information they need to work out the area. It is likely they will say you need to know the length of two sides. From this discussion students are to draw their own triangles on the gridded mini-whiteboards however, only one of the sides can be along the gridded (to make a non-right angled triangle). Now challenge to students to find the area by measuring the two sides. Students will try to use the rule “area is equal to half base times height” if they do use that rule ask students to draw a rectangle, they will be unable to draw a rectangle. Bring the group back together and ask them why this rule doesn’t work. Students may work out that the height is not any of the sides, from this have a group discussion on why that is. Progressing from this discussion have students investigate further on what kind of rectangle will work for non-right angled triangles. Students should identify that there needs to be four right angles to have a rectangle, and that was what was helping us to find the area of a triangle, so how can we make a right angle in a non-right angled triangle? Let students try and make rectangles from the triangle. Once students have tried various rectangles and some had gotten the idea, draw this up on the main board. 

Ask students whether this rectangle is twice the size of the triangle?

If students can’t see the two smaller triangles prompt them with questions such as these

*What is the area of the left-hand rectangle?*

*What is the area of the left-hand part of the right-angled triangle?*

*What is the area of the right-hand rectangle?*

*What is the area of the right-hand part of the right-angled triangle?*

(nzmaths.co.nz)

From that students, should figure out a way to illustrate it to show similar area



This drawing clearly shows two triangles with matching rectangles, have the students prove that this drawing works for any non-right angled triangle but getting to draw various other triangles to show to a peer. Then from that have the students write down what they discovered about heights of triangles in right angled triangles, non-right angled triangles and the rule for the area of triangles. After students, have written their thoughts down, ask students to form groups of 3 to read what they have written to each other, students are told to report back one thing their group said that was the same and one thing that is different. This allows students to reflect on their own learning and the learning of others. This lesson builds up for the final lesson in finding an area of a triangle.

I have chosen this lesson mostly around discussion and using language to explain mathematical content is because the use of language is critical for students to understand mathematical concepts. Having the students to find their own meaning in what they are seeing and learning and contrast it to other student’s ideas extends their reflection and allows students to discover and learn from each other as stated by Reyes, M. R., Brackett, M. A., Rivers, S. E., White, M., & Salovey, P. (2012) “Engagement is changing, can’t hand everything to students, they have to ask it, they have to invent it. They have to be the center of everything” this means that students need to discover, uncover, and investigate their own thoughts by themselves (and guidance) and discussion is a perfect one of reflection on own thoughts and others to develop learning. This brings in culturally responsive pedagogy; ako where students are enabling themselves to have a voice and become a teacher as well as a learner and whanaungatanga - building relationships. Kiefer, S. M., Alley, K. M., & Ellerbrock, C. R. (2015), discuss the importance of peer relationships in academic achievement of learners. The study states that peers offer different types of support to the learners, emotional and academic support. These can be seen in the example where the student is finding the current content difficult and is too shy to ask the teacher for help, this is where they can turn to their peers and ask them for support on the content.  This approach to this lesson was from a cross-curricular perspective using English as much as possible to create understanding. Ward-Penny, R. (2010) states that a learning maths in a cross curricular approach is authentic and more motivation for the learners, and helps learns to make sense of mathematical ideas. Students who do not enjoy mathematics as much are enabled to understand key learnings through other areas, in this case English. This lesson is a stepping stone towards building students’ knowledge, this prepares for next lesson from their own language to mathematical languages such as the formula for finding the area of a triangle.

Lesson 4 will comprise of students explicitly using the rule for finding the area of the triangles and allow me to find out who is still struggling with this concept.

 This lesson will start off with a Kahoots! A online quiz game that requires devices. Students are having the Kahoots! Nicknames as their real name as this can be used as an informal assessment later as this online website stores data on student’s answers. The questions will be a recap of what they have learnt previously such as types of triangles, find the area to this rectangle/triangle. This lesson will then extend towards a discussion on the rule for finding the area of a triangle (this was the last question in the kahoots)

After a think, pair, share on the rule of finding the area of a triangle students are to individually work through a [worksheet](https://drive.google.com/file/d/0B1_EYWvkdKv1eXZxcFVzTlJHOU0/view?usp=sharing). This work sheet develops from basic questions with right and non-right angled triangles with various rotations, students are asked in the second part of the worksheet to sketch triangles from word descriptions. By doing these worksheets students should have concrete ideas on finding the area of a triangle.

The second task in the lesson is to give students find triangles within the classroom and take the dimension of the triangle down, students are given the task to find at least 3 objects that are a triangle and to find the area. This is an indirect approach to solving application style questions that serves as an extension to students.

Extra for experts: This is an [online game](https://www.studyladder.co.nz/games/activity/area-of-triangles-13135) to find the area of a triangle

The justification of this lesson approach is to bring a more exciting way to learning and doing mathematical problems, by introducing Kahoots! Students feel more interested in this lesson although it is a lesson on solving problems. Bringing technology motivates students in learning (Niess, M. L. 2005), and this motivation can extend their academic achievement found by Nichols, J. D., & Hall, N. (1995). Teaching with the use of technology also creates the idea that doing mathematics isn’t always pen on paper work as students are 21st century learners who are encouraged by connecting with others. This is also the last lesson in finding an area of a triangle, and bringing something more light and interactive for the students reminds them that math isn’t boring. Based on the research by Ruffell, M., Mason, J., & Allen, B. (1998) which found that a dominant of students had a bad experience in learning math. By having more collaborative work, and the use of technology this bad experience is minimized and students will be eager to learn mathematics. This lesson also extends their learning to the real world, often students will ask “what is the point of learning this” or “how can we use this in real life”, the activities in this lesson allows students to realize that math can be all around them, even something specific as finding an area of a triangle.

Overall the justifications of my lesson were to extend students motivation in learning mathematics as well as minimizing misconceptions in finding the area of a triangle. Having group and pair work maintains a healthy classroom environment through relationship building which is backed up by Zins, J. E., Bloodworth, M. R., Weissberg, R. P., & Walberg, H. J. (2004) findings, which states that a safe learning environment creates better academic achievement for the learners. Motivation is increased using different types of learning strategies such as the use of materials and technology which also is positively correlated to academic achievement in learners.

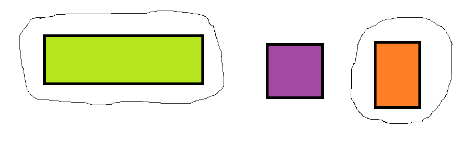
Possible Te Reo vocabulary usage in these lessons

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| --- | --- |
| Te Reo | English |
| Tino Pai | Great stuff |
| Ka Pai | Well Done |
| Kua mārama/He patai | Understood?/Any questions? |
| Kua mutu? | Finished |
| Titiro mai | Look this way |
| Whakarongo mai | Listen this way |
| Huri mai | Turn this way |
| Me mahi taki(rua) inaianei | Work in (two) now    - (Number) |

Misconceptions and misunderstand is prevalent in teaching geometric properties, especially triangles. Throughout my lesson sequences I have tried to inject various misconception that may arise. Many misconceptions of triangles stem from prior learning of geometrical figure and wrongful understanding of definitions. Based on the research by Cutugno, P., & Spagnolo, F. (2014) they found that 45% of students did not understand the term geometrical figure. This is because they had a conflict of mathematical language and everyday language. Students also generalised triangle properties as the sides having the same length of sides and angles, this was shown in their research stating that 31% of learners see triangles resembling a equilateral, with 59% would draw a equilateral triangle when told to illustrate a triangle. This research shows that students have a strong mental image of what a triangle looks like. Furthermore, the research also found that the height of a triangle is always a vertical line, and that students do not recognize the height if there is no horizontal base. These misconceptions were taken into consideration during the lesson sequences as there were multiple discussions of finding non-right angled triangles and the height of the triangle where the there are no points that have a 90-degree angle. It was stated that some of the misconceptions they found is that learner’s thing triangles have one point at the top and two points at the bottom, with the bottom of the triangle is flat. Problems that were given did not always contain triangles that were “rigid”, problems were given that had triangles rotated or tilted to show that the triangles did not always the be the same orientation, the triangles given in the lesson were of a variety of sizes to also enforce the idea that a triangle can be any size or rotation. Crowley (1987) found that a discussion approach to learning helps students to understand mathematical concepts in a more encouraging way by having the students talk about it in an informal way. Other research such as Orhun, N. (2004) also reported by clearing the misconception of “rigid” triangles to get students to flip their books around (In this case mini-whiteboards) to analyze the shape.

A common misunderstanding among students is the new array of vocabulary used, students are often confusing, Mitchelmore & White found that when children are overwhelmed by the specialist vocabulary and are not able to put into standard language they often become disengaged with the topic preventing learning. The research states that encouraging the learners to talk through mathematics by encouraging group work and discussion provides the learners the opportunity to use this vocabulary in a less formal context. This research is also backed by research done by Crowley (1987) which states that a discussion approach helps learners build confidence, empower them, and allow them to consolidate their understanding.

A misconception that is also clarified is to do with rectangles, students will recognize that rectangles are long at one end and short on the other, and would not see the purple shape below as a rectangle.



This is a misconception, all the shapes above are rectangles including the square, students would have be addressed this misconception in the previous unit however, so it was not made explicit in the lessons sequence. Moreover, if I was teaching geometry without knowing their prior knowledge then I would include a discussion on how a square is a rectangle, this links back to the first misconception which mentions to a misunderstanding of the definitions of geometrical figures.

The learning sequence has been made accessible to the students as it incorporates culturally responsive pedagogy. I have used ako as student reciprocate learning, whanaungatanga through pair and group work, manaakitanga and Tangata whenuatanga as I have acknowledged, respected, and valued who the students are and where they come from and, through deliberate and reflective practice, building on what they bring with them to the learning setting as well as taking students concerns and ideas seriously and valuing their culture in subject content and learning contexts. The context the students are working in relates to the student’s environment, I have done this through allow students to investigate and explore their surroundings to see if they could find objects similar to their learning and find the area of the object they have found. This is linked towards the Ministry of Education (2008) curriculum document which states teachers should enhance the relevance of new learning by making connections to prior learning, experience and setting.

I have also tried to make my lessons appealing towards students of Maori and Pasifika descent by trying to incorporate various vocabulary into my lessons (can be seen in the table above) such as using various phrases for classroom management and praises. I have also used Te Reo as a way of delivering instructions “Me mahi takirua inaianei” which means work in twos now. This is used multiple times as my lessons encouraged group work and discussion, this is also based of the research Vygotsky (1978) conducted, Vygotsky's learning theory is based a social constructivist view where “talk is a tool to develop thinking”, having multiple opportunities for the students to discuss their ideas, their findings and as well as their answers students are developing their thinking through the use of a language another important aspect in understanding mathematical concepts Powell, K. C., & Kalina, C. J. (2009). Furthermore, group work and discussion promotes motivation within the classroom, according to So, H. J., & Brush, T. A. (2008) working collaboratively allows students to become more motivated in the classroom which in turns increases academic achievement.

 The lessons have also been made appealing and accessible through the use of technology, students were given the opportunity to play an online math game as an online quiz (relating to math) using the devices they had on them. As students are 21st century learners, they would want to be able to use the tools that are available to them in learning Garofalo, J. D. (2000).

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